23. $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n} = 1 + \frac{(x-1)^2}{4} + \frac{(x-1)^4}{4^2} \dots$

$a = 1$ sum $= \frac{1}{1 - \frac{(x-1)^2}{4}}$

$r = \frac{(x-1)^2}{4}$

$|x| < 3$

$-3 < x < 3$

$|x+2| < 5$

$-5 < x+2 < 5$

$\sqrt{x^2} = |x|$

if $\left|\frac{(x-1)^2}{4}\right| < 1$

$-1 < \frac{(x-1)^2}{4} < 1$

$-4 < (x-1)^2 < 4$

$0 < \sqrt{(x-1)^2} < \sqrt{4}$

$|x-1| < 2$

$-2 < x-1 < 2$

$-1 < x < 3$ JAM.

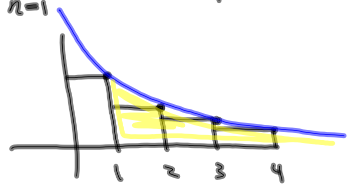
inc

Feb 9-9:46 AM

9.5a More Test for Convergence

The Integral Test

$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} a(x)\,dx$ behave the same



Feb 7-9:43 PM

Does the series converge?

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ converges

$\int_1^{\infty} \frac{1}{x\sqrt{x}}\,dx$ converges to 2

$\lim_{b\to\infty} \int_1^{b} \frac{1}{x\sqrt{x}}\,dx$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $\lim_{b\to\infty} \int_1^{b} x^{-3/2}\,dx$

$\lim_{b\to\infty} -2x^{-\frac{1}{2}}\Big|_1^b = \lim_{b\to\infty} -2\cdot b^{-\frac{1}{2}} - -2\cdot 1^{-\frac{1}{2}} = 2$

Feb 7-9:51 PM

Harmonic Series and P-Series

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \text{if } p > 1, \text{ series conv.} \quad \text{if } p \le 1, \text{ series div} \quad p \text{ is a constant}}$$

$1 + \frac{1}{2} + \frac{1}{3} \dots$

$\sum \frac{1}{n^1}$ $p = 1$

Feb 7-9:53 PM

Limit Comparison Test

$\lim_{n\to\infty} \frac{a_n}{b_n} = L \quad 0 < L < \infty$

If a_n & b_n ~~grow at the same rate~~

then $\sum a_n$ & $\sum b_n$ behave same

Feb 7-9:54 PM

Do the following series converge or diverge?

$\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2} \qquad \sum \frac{2n}{n^2} = \sum \frac{2}{n}$

diverges by #lct

diverges

grow at same rate: (yes)

$\lim_{n\to\infty} \frac{\frac{2n+1}{(n+1)^2}}{\frac{2}{n}} = \lim_{n\to\infty} \frac{(2n+1)}{(n+1)^2} \cdot \frac{n}{2}$

$= \lim_{n\to\infty} \frac{2n^2+n}{(n^2+2n+1)2} = 1$

Feb 7-9:56 PM

$\sum_{n=1}^{\infty} \frac{1}{2^n - 1} \qquad \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots$

converges by #lct

converges geo $r = \frac{1}{2}$

grow same yes!

$\lim_{n\to\infty} \frac{1}{2^n-1} \cdot \frac{2^n}{1}$

$\lim_{n\to\infty} \frac{2^n}{2^n-1} = 1$

l'hopital's $\frac{2^n \ln 2}{2^n \ln 2} = 1$

Feb 7-10:01 PM

$\sum_{n=2}^{\infty} \frac{3n+2}{n^3 - 2n}$

Feb 7-10:01 PM